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ANDOM NUMBERS FROM? A RA 101

SOURCE

?OPERATED BY UNIVERSITY OF PUERTO RICO UNDER CONTRACT

MO. AT ?40-11-1829 FOR U, 5. ATOMIC. ENERGY COMMISSION

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Random Numbers from a Redioactive source

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and

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* Submitted to the University of Puerto Rico at Mayaguez, Mayaguez, Puerto Rico in partial ful~ filiment of the requirements for the degree of Master in Science (Nuclear Engineering) -

Work Performed at the Puerto Rico Nuclear Center, Mayaguez, Puerto Rico.

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Mayaguez, Puerto Rico

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by

Amoldo J. de Hoyos

A thesis submitted in

partial fulfiliment of the

requirements for the degree of

Master in Science

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Chairman Graduate Council Date

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I wish to express my gratitude to the many persons who collaborated in this work but especially to dr. Dennala 5. Sasscex for his aavice, help and continuous encouragenent in the performing of the whole thesis.

T also wish to thank 411 the personnel of the computer Genter of the college of Agricultural and Mechanical arts and especially to Mrs. A. Kay and Mrs. A. I. de Alearin for theiz help in programming the conversion and test of the random nunbers.

This research was performed during the period when the writer held an international atomic snergy agency

fellowship at the Puerto aico Nuclear center.

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The statistical fluctuations in the disintegration of

@ ©8197 source meusured by Kadiation counter Laboratories! 54 multichannel analyzer and a ganna ray scintillator Getector were used to produce a sequence of uniformly ?istrsbuted binary random digits. this binary sequence

was then converted to 4 decimal sequence and seven tests

Of Fandonness and unitormity of the distribution were

appliea.

contradict the hypotheses that:

*) The distribution of the decimal digits tits the

uniform distribution (frequency test)

») There exists no correlation of association anong

She digits of the sequence (auto-correlation,

mean square disference and serial tests)

©) There exists no abnormal clustering or dispersion

among the digits (runs tests)

©) There is not a favored tive digits arrangenent

(poker test)

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?Tables of the uniformly distributed binary and decimal

random digits are presented. ?These random numbers or pseudo-

random numbers generated from these may be used in statisti-

eal sampling and in Monte Carlo calculations.

The method presented in this thesis to generate a sequence of random numbers compares favorubly to other methods that are reported.

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A table of uniformly distributed binary random

nunbers.

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A table of uniformly distributed decimal random

numbers...

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Table

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ergopuctros

?The purpose of this work was to generate random numbers by using the statistical fluctuations in the @isintegration of a radioactive source.

A sequence of numbers is called random when there exists no dependence among its menbers. That is, each nunber gives null information in guessing other nunbers.

The Monte Carlo method consists of solving a physical or mathematical problem by using random numbers to simulate 4 Fandom process that is directly or indirectly related to the original problem.

Beside their use in the Monte carlo method, random punbers are widely used in statistics as « safety rule against bias in sampling and also to give validity to teste performed in the design of experiments.

Random nonbers are generated by two general ways atithnetical and physical procedures. arithmetical proce- @ures consist of iteration methods based upon a mathematical formula that generates pseudo-random nunbers. These number s are only random érom the point of view of some specific application or by passing several statistical tests for

randomness, Physical procedures consist of a physical

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device (1.

- dice, a roulette wheel, a radioactive

Source), used to generate a sequence of random numbers.

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REVIEW OF LITERATURE

In 1949 J. von Neumann said "Anyone who considers arithmetical methods of producing randon digits is, of course, in a state of sin" (1). However, up to now the most common way to generate random nunber was arithmetical: The congruence method.

Why, 4 we are convinced that our deterministic mind

can not conceive ways to generate random numbers, do we insist

on the arithmetical process? The answer in 1959 of the

International Busin

Machine Corporation to this question

is that "Past arithmetical procedures (congruence methods) de exist whose results, though of course not random, never

the 1

do furnish a satisfactory gubstitute". In addition

they are opposed to the generation of random numbers by

physical proc

feo because ?Nature tends to be systematic,

90 the construction and maintenance of a mechanical or

electronic device - a perfect roulette wheel - is not at

ali cheap or easy for practical necessities" (2).

% report of Nac Laren and Marsaglia in April 1964 (3)

concerned with the suitability of the arithmetical processes

---Page Break---

Of generating random numbers showed that the sequences of

pseudo-random numbers generated by arithmetical processes could pass many tests for randomness and still be unsati:

factory when used in Monte Carlo calculations for some

practical problem:

they suggest finally that the most

suitable sequence of random numbers is that obtained from

@ table of random numbers, which itself is generated by a
physical process.

An answer to the second argument of the IBM Corporation
which was the same as that of Brown from RAND Corporation
in 1949 (4), concerning the difficulties of generating
random numbers by using a roulette wheel is that a roulette
wheel is not the only physical process that can be used to
generate random numbers, in spite of the fact tht RAND
Corporation used it for the construction of a 1,000,000

random digits table (5).

Perhaps the strongest argument against generating

random numbers by physical processes is, as J. V. Neumann Pointed out in 1949, the practical need of reproducibility,

that is of repeating the calculations exactly.

SIE eens that there exists physical phenomena where

randomness is essential. Nature apparently is not completely deterministic.

---Page Break---

If the physical process is incorporated directly into @ computer, as it generally is, then this criticism my ?be answered by noting that it is always possible to print out a given sequence of the computer input random numbers and use this sequence to repeat the calculations. Another Possibility is to look for systematic erros in the actual running of the program by performing statistical analyses of the results.

Kahn in his research memorandum of 1954 "application

of monte Carlo" (6) claims that it is impractical to print

@ sequence of random nunbers generated by a physical proce!

because of the limited menory and input - output capacity

of computer maching

However the practicality of using

@ table of random numbers (generated by a physical process)

for Monte Carlo calculations was demonstrated by Mac Laren

and Marsaglia in April 1964 (3). They used different

ctions of the computer menory alternately and as a buffer,

?nd concluded that a table of random numbers is the most

suitable vay to generate a sequence of random nunbere

Since it was found that in several cases pseudo-randon

numbers generated by arithnetical process were unsatisfactory

and also, since modern technigu:

make the use of random

---Page Break---

numbers generated by a physical process practical; it is

now reasonable to start thinking again of better methods

f generating random numbers by using physical processes

cal proc

to.

jerate random numb

to

Several investigators who used physical proces:

obtain tables of random nunbers are; Tippet who used census

Feports (7), Kendall and smith who used a mechanical roulette

wheel (8), Hamaker who used a rolling 10 sided priem (9), and

the RAND Corporation, which produced the largest and most

uused table by means of an electronic roulette wheel (5).

Research on the u

of a radioactive source to generate

Fandom nunbers was done by; J. Von Hoerner who generated

sequence of randon binary digits by considering the posi-

tion in time of @ flip-flop activated by a radioactive source

counter (10), and also by M. Isida and H. Tkeda who generated

and incorporated into a computer a sequence of decimal random

digits obtained from the last digit of a pre-set time radioactive source counter (11).

---Page Break---

THEORETICAL CONSIDERATIONS

Experimental observations of a radioactive source show that the radioactive decay occurs at random and at independent monents of time.

It is known that radioactive decay can be represented by @ Poisson distribution (12) which tends to the normal @istribution with increasing counting rate (13).

In order to use radioactivity to generate random Runbers with sone specific distribution it is convenient to convert the non-uniformly distributed random numbers obtained by counting a radioactive source, to a sequence

of uniformly distributed random number

?This is accomplished by comparing the successive nunber of counts of a radioactive source and

jigning a one or a zero to the comparison

depending upon certain criteria*. By this process the

original non-uniformly distributed random numbers are

converted into a sequence of uniformly distributed randon

binary digits, which are then transformed into a sequence

of uniformly distributed random decimal digits.

?The advantage of this method over the other physical

methods mentioned earlier (10, 11) is that the final aistri-

jee procedure page 9

---Page Break---

bution of random nunbers is not affected by the variance of the original distributions®.

I£ one is interested in incorporating the radioactive

Process directly with a digital conputer, then a slight

variation of this method gives faster and therefore better

results. For example, the fact that in two successive

Gisintegrations there is the same probability that one

takes longer than the other can be used in connection with

an electronic clock to feed a sequence of uniformly distri-

buted random binary digits directly to a computer.

jasion of results page 26

---Page Break---

EXPERIMENTAL PROCEDURE

Generation of the random numbers

A Radiation Counting Laboratories! 512 multi-channel

analyzer with a Tally tape perforator and a gamma ray scintillation detector were used to measure the activity of a Cs'97 source of approximately 1 microcurie*.

The analyzer was used as a preset time scaler in the automatic mode. The information in the channels (counts Accunulated during 0.1 seconds) was readout in perforated

tape. This tape was converted to 18M cards.

Conversion of the original non-uniformly distributed xandom numbers to binary and decimal sequences of uniformly

distributed random nunbers**

A sequence of uniformly distributed binary random Aigits was formed whose mth term was defined as 0 if Nem was less than Nong, OF 1 if Neq was greater than Nemes, where Nzm and Neamt, were successive channel counts. Each exclusive set of ten binary digits was then converted to three decimal digits, producing a sequence of uniformly

@istributed decinal digits.

?See photograph next page.

?The conversion was performed by an 15M 1401 computer.

---Page Break---

?Szequnu wopuex TeuTbT IO 9y3 99ez0UI6 03 pasn 3ueudENbs

SzequowoTduco pue zozkteue TouuRYO-T4 TMU TY aya Fo YdexBo30ya ?1 eanbTa

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ry

Tests for randomness o% the decimal digit

Im order to measure the independence of the decimal

@igite and the uniformity of their distribution, four blocks

©£ 1000 decimal digits each were obtained and tested for randomness using lags 1 through 10. tag 1 is defined as

picking every consecutive number Ny, Ngs+++) Njggg of the

Sequence of decimal digits in a block as the random quantity,

Jag 2 as picking every other number N,, Nq,--

foe Nas Ngeeey

Moo Of the sequence and go on.

Pive of the seven tests used are reported in a very

recent work of Carlsen (15). They are as follows:

1. The frequency test which consists of calculating

chi-square

x= 1 2 (¢, ~ 100)

" "Too" s80 #

where £, is the frequency of the decimal digit i (i = 0,

1s++4, 9) in the block. The average and variance are also

calculated, the obtained values are then compared to the

expected values.

2. The serial test which consists of "binning" the

numbers into a 10 x 10 matrix. A 1 is added in row i

?The tests were performed by an IBM 1620 computer.

---Page Break---

1

column j, when digit i is followed by digit j (i, 5 = 0,

Ays04, 9)+ The expected result would be 10 in each position

of the matrix. The following x* is then calculated.

ent 2 2

xe= (#,5 = 20)

r

10 i, jo

x2 - XP should be distributed as a chi-square with SO DP.

The expression z = (2(x$ - x#))'/x - (2 x 90 - 1)'/2 48

sed as a normal deviate with unit variance and the observed

valu

of Z are then compared to expected values.

3+ The auto correlation test which consists of

calculating the auto correlation coefficient

oe Pa,

sar ah

for h= 0, 1, 2,..., 10 where ¥, is the mth term in the lock and comparing ch to expected values.

4. ?The runs test number 1 which consists of finding

the runs above and below the mean. To find the runs above

and below the mean a sequence of binary digits is formed

whose mth term is defined as 0 if Mis less than 5 or 1

if M, Je greater than 4. A subsequence of k zeros (or ones

bracketed by ones (or zeros) at each end forms a run of

Jength k. Runs aze counted and compared to the expected

values.

---Page Break---

43

5. The mean square difference test which consists

of calculating

2 1000)

Me 5 (Ny ~ Nag)?

2000 m1 | ®

where d= 1, 2, 3, 4, 5, 10, 100, 101, and comparing m to the expected values.

In addition to these tests, another runs test and a poker test recommended by Brown (16) were used.

6+ The runs test nunber 2 which consists of finding the frequency of runs in the decimal sequence, and comparing this values to expected results.

7. The poker test which consists of scanning the Gecimal digits in groups of five digite each, simulating

Poker hands, and finding the frequency of seven classes of

hands; busts (symbol abcde), pairs (symbol aabed), two pairs (symbol aabbc), three digits alike (aaabc), full house (symbol aaabb), four digits alike (synbol aaaab) and all

five digits alike (symbol aaa). Results are analyzed

using @ chi-square

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RESULTS

?The following results are based on a sample of 4000

decimal digits. averages are referred to the four blocks

Of 1000 decinal digits each.

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S0°02 60°02

62-60 19°6t

66°61 26°6t

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9 WE"HT OfL-tt LoT-9T EoK-ot E6S-9T zee-oT OOkST JgL-oF

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Table 6. Runs test ©

Length of ru 2 3 5 tag

Observed average 821.00 77.50 7.00 0.75 0.00 1

number of runs 626.50 76.25 5.25 1.00 0.85 2

804.75 80.50 9.50 1.50 0.00

803.75 79.25 10.25 1.00 0.50

800.00 86.00 8.70 0.50 0.00

810.25 79.75 8.50 1.25 0.00

810.25 80.00 9.25 0.25 0.25 7

863.75 75.75 6.25 1.25 0.25 8

£12.00 77.75 10.25 0.50 0.00 9

602.75 63.50 8.75 1.00 0.00 10

Bepected

punber of runs 610.00 61.00 8.10 0.61 _0.09

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3893 soya *L oTquL

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DISCUSSION OF RESULTS

Frequency test (table 1

An inspection of table 1 shows that all the digit

frequencies are close to the expected value of 400 for

@ random sample of 4000 uniformly distributed decimal

digits. Five of the digit requencies are above and five

Below 400. The digit zero has the highest frequency 432,

and the digit one has the lowest frequency 372. the even

@igits have a total frequency of 2017, and the odd digits

have a total frequency of 1983.

?The chi-square for these digit frequencies has a value

of x} = 9.165 which is smaller than 16.92, the critical

value of the chi-square for 9 DF at the 95 °/o confidence

jevel. The blocks 1, 2, 3, 4 of 1000 digits each, give

the respective values for the chi-squares of xf, = 12.02,

X72 7 10.86, x8, = 9.44 ana x3, 12.64. The average of

these chi-squares is ¥7 = 11.24. All of the chi-squares

have values that are less than 16.92.

?The average of the digits is found to be 4.48 which

' somewhat lower than the expected value of 4.5 for a

uniform distribution in the interval from 0 to 9. the

averages of two of the blocks were below and two were

above 4.5.

---Page Break---

It is interesting to note that the averages reported

by Carlsen (15) of 100,000 uniformly distributed pseudo-

Fandom nunbera obtained by congruence methods and Brown (16)

Of the 1,000,000 digits of the RAND table (5) obtained by using an electronic roulette wheel are both below the expected mean for the corresponding uniform distribution.

?The observed variance was 0.

a value that is lower but close to the expected value of 0.833. Based on the law of large numbers all of these results should improve with increasing size of the sample. These tests give no evidence to reject the hypothesis that the decinal sequence

is uniformly distributed.

Serial test (table

The object of the serial test is to indicate the tendency of given digits to be associated with any other

digit. The values of z for the different lags are Le:

than the value 1.96 of z for the 95 °/, confidence level.

?This test indicates that the hypothesis of mutual associa-

tion of digits is rejected.

---Page Break---

Auto-correlation test (table 3)

The results from the auto-correlation test indicate
that the average values of ch for different h (h = 0, Ly..,
10) and different lags (1ag 1, 2,..., 10) are somewhat
lower but nevertheless close to the respective expected
values. It seems reasonable to assume that these values
Should be low due to the fact that the observed average
value of the digits is lower than the expected value as
is shown in the frequency test.

?This shows that there exists no significant evidence

of correlation anong the digits*.

Runs test 1 (table 4}

Runs above and below the mean give a measure of the
tendencies of the digits to cluster or disperse with respect
to the mean.

The observation of the values presented in table 4 clearly show that there exists no significant evidence of abnormal clustering or dispersions of the digits with

respect to the mean.

?Notice that correlation is not a measure of independence but only of linear dependence (13).

---Page Break---

Mean square dissezence test (table 5)

The mean square difference test provides a measure Of the association among the digits.

?Phe average values found of M for the different @ (a © 1

2, 3, 4, 5, 10, 100, 101) and different lags (lag 1,

10) axe also lower but close to the respective expected

values. As with the correlation test the observed values
©f M should be low since the average of the digits is less
than the expected. Therefore there exists no significant

evidence of association among the digits.

Runs test 2 (table 6

?The consideration of table 6 gives an indication of
the degree of clustering or dispersion of the digits among
thenselves and shows that this type of anomaly is unlikely

to be present in the sequence

Poker test (table 7

Table 7 gives an analysis of favored arrangements
Of five digits, it shows that the values of the frequencies
for the different hands (busts, pairs, two pairs,..., fives)

and different lags (1, 2,..., 10) except for lag i are close to the respective expected values.

---Page Break---

Fitness to the expected results is tested with a chi-

square, and it is shown that 411 the values of the chi-squares

except that for lag 4 are leso than the critical value of

12.59 for the chi-square of 6 DF at the $5 °/, confidence

level.

The anomaly which occurred in lag 4 is easily explained

by noticing that this lag has one very unlikely but possible

artangement of fives (aasaa). the presence of this arrange-

ment gives the main contribution to the large chi-square

obtained.

Advantages and disadvantages of the method used in

this thesis to generate random numbers

The arithmetical procedures have the advantages that

She munbers are eusy to generate and easy to reproduce, but

have the disadvantage that they cannot generate random

nunbers.

Peeudo-random nunbers generally pass various tests

for randomness, but this only proves that we do not know

She type of dependence established in their generation, and

sherefore we are not able to test for it. Nevertheless it

ie true that in some problems some types of correlation

---Page Break---

may not affect the reaiits and in those cases arithmetical

procedures may be more desirable,

The methods using ¢ radioactive source to generate

Tandon nunbers have the advantage over other physical methods

that the necessury equipment is eusy and cheap to maintain

and that mechanical devices which gives special tendencies

in the sequence of random nunbers is eliminated.

AS mentioned earlier the method ui

2 in this thesis

when conpured with the two other metiods taut uses a radio-

active source to generate random numbers has the advantage

that it does not depend on the variance of the counting rate. M. Isida and H. rkeda for example who used the last

@igit Of a count to generate a sequence of decimal digits

must Keep the average counting rate (variance) greater than

2

in (a)

im order to have 4 relative error in the uniformity of the

@iatribution leas than a, where a is the naxinun difference

of frequencies among digits and s. von Hoerner used the

false assumption that the average is in the middle between

88 oda and even number. the error in the uniformity of the

distribution is inversely related to the variance.

---Page Break---

Perhaps the strongest assumption made in the methode

that use 4 radioactive source to generate random nunbers

i that the process of radioactive decay is stationary with

Fespect to tine, whereas the activity decreases with time

by @ factor of exp (At). the factor of decrease exp (~2t)

for the method used in this work (Cs'97 source, 0.1 second

counts) is

Ine z

ep (- :

st = 0.9999999999 4 1

30 * 365 x Gh x 360 Io

?This shows that the assumption that the proce:

is stationary

is a very good approximation.

---Page Break---

CoUCLUSION

?The method used te ges

rate random nunbers in this
work appears to be better than the other physical method
Feported in the literature. The physical methods have the

advantage over the arithmetical proce:

in that they
generate random numbers.

?The sequence of uniformly distributed decimal random
nunbers successfully satisfied all of the seven tests for
randomness performed.

Tables of unitormly distributed binary and decinal
fandom pumbers are presented in appendix 1 and ¢ and are
available also as iN cards that can be used to draw random
samples in statistics and these to generate sequences of
Pseudo-random numbers for Monte Carlo calculations as
suggested by Mic Taren and Marsaglia (3). these results
suggest to one who wishes to make use of the Monte carlo

method that the construction of a device to generate random

pumbers using the disintegration of a radioactive source

might be incorporated directly into a computer with advantages.

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Q)

(2)

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(a)

(ae)

(33)

(a4)

(as)

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APPENDIX 1

A table of unifornly distributed binary random numbers

9121001 111 000G010:001 v20000110100000111 10011110001

910101100001 10101001101 00011000001101100101 101011

Lit oo11 1002110011 100110100111 1001 101001000100010

1110000100111001101 1000101 co010011oL01 00101 Lou

9110011011000001111110100000co010100001 101 100001001

G9OLOL0OL010L00111112201 02 1100121110111 1oo0LOLOrT

2010110111010171000011000011111000011001000001 10010

90111101010011110100011011 0¢1 1000210010011 101008

2400110110011001111011111001010111110001 0101 101001

901111110109101000110110101120011100111011 11000101

1011001 101100001000100101101100101101000001 10C0001

10001001100100100001111011111100101010 0101 00100011

000001110001 00010001001 1141c11121001001 L1111 000010

91101111101000001101011 0001 1010000111101 11000119

2101 1001010700100010011 0011¢00010010101100101 10100

911101100000010001000101101¢10101100001 00000101001

4100001100101 211101120011 00011101101001 00111 10081

9001110010190100110111 00001coL0001010011111 0101111

9021110001 100000000011L0101CL011110111 0001 01011101

911101111010100001001 0001 00¢10101111101 11100100100

4?001010000001110010010000 1¢11101110010100010 10000

10010000011111101111011101010001000 0101 00100001111

92201100110100010001111010111012 0100001011111 1110

99010000000" 10101101011 0000c10000110001000000001111

100010001010010000100010114¢10001111001011010100000

8101000100101011011110101011120101001 11010100001 19

9111101110090001001 100111011C011001100001011 1001109

94410010100°000111 ic101010¢101011101100101011 10110

120100100111011011100001010¢11 000101000101111111010

200010111010100101111101100110100101001010010100100

911001110100010030010101001 110000001011 1011110000

31010C001 1001010101 10901110101 10011 50001 tL LOD ToF

991111111101 101c00010100100c0L000011101 10110100010

0110010000001 1011001110011010000000101010101 100100

12010011101011200000011111110201 100000111101 L01110

400111111101201111101000111¢10010111001101 00010111

1011100100011 10001 :o000010111101100001 100011 1001 10

9011110100010110001 101 0000001201 1010001 100011011101

gtoL010101 0101 1101000001 10¢110010000110000100 1100

920101011007100101010001010010411011010001 1100101 |

110010101011000011 100106001 100100001 101111010011 10

42111010101011111001010021 1c00000101 1000110111 1010

9100101 1010100001100110000 0C100111 11010101001 10101

400001010110110001010011100C1121001110100000000011

110011900011011000 10001 1010101 10100100101111111110

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% table of uniformly dictributed binary random numbers (cont)

tettocoio01) 11001107111 1101100001101 1601 0100010010

1011001101703 1011010010101 Loot Lortcionootol tiitt

{11111 LoL01¢00010101 1100010119111 00¢000001 10001010

}eootototvicios 1111011 161G0100011 101 111101100101 01

Go00oco0001 900101004111 1000C1 1010101 10010111 101101

4011 12CO01O0110111011101111220101 1001 10000001011

920101000 1070090111 14900001 111C10000001 00001101000

9010001110011101100c000100010101111111 10001100000

$00000000111110011100011100100021111100100101 10011,

0140011110071001 1001101111 0¢1 00LoL OC LOLUNL TOON LTT

Q1110011001) 1o101010010¢c01C1 v0100101000011 1100101

1001C001119010111010011011 104400101001 10000011000

9100110101 00110110111001001c1 100001 0100000111101 00

Yorrocoacorriootttt iii itoIcLAtLoLolttorolLol ioe

41100111000 100100L101112111C01111 LocoLoL tLOOLLtT ET

$eLLo1oL010010101 110001111001 1101000201011 111000

{4000t100007011111111110011co101100101101 100111001

990010100101 110010001 101010101 00001 10010111 1000001

2011000101 1M101C0101010G001110010L0C001L LI LALO IL

9010001 110111110011 101 0cco0coLcL001111 101101010000

1900160C00170100111¢11101001001010111011 1010010000

d401040110720101011110C001¢C100100010011 1011000100

900101100101010010110011111100001000101 0000001009

401111 1000111100000100c000c00101 000000001 10000000

190011011 109000111101 1010111101001 1000000000101 tt

00040014000 1000101 00001 1actu10011011110201 000001

9041100101191 10104 00110101 100100011 10101 101001110

{90L010111001110000100l 00001 101 1oUL0L1 10101011011

9000101 1011101 LooLo11010L00210101001 11 LouLOLOL TOO

011 OLorL0FOL 10110000111 20C1200111 1001111 10000111

{oLoct 101s ro1t011 100001 101121111111 1001100110010

41110¢001001111001000u100111111010001111 0000001001

SoLorcoLtL 1 YOOL10111011 1000121020001 LoL Lt tool LAI tt

30010100001"01101 100001 1010C01111001 0011010010011

901111011101 1000100101 00010ct0011 1000too101 1010111

Botot140111"01210111101111000010010011001 1010100

9101101 100000011101 111001101010111101110001111010

$11 4010010171110001001010 10C10111001 11000000001 111

$61010100001000110111010001C01 10011 LonLo1L LooLorot

901001011111001 10101001 L001¢001 0001112100001 111104

91001100111000100011110010400000100001100010010 00

9100001 1100110010001 101001Coo0t 10010101 1101110010

9odd001011 19011 100132 1001 11C1L111110001 LoLt LocooOL

$10101101019010110000100100¢2110100010110110101000

1001011110010111010110110 01 1001010011001 1011001111

---Page Break---

A table of unifcrnly aiotesbuced binary random nusbere (cont.)

!10000000101 10% : 100001 10001111 100110010011 201001,

0101111010111091 1101001 ¢o01co0c1 101 1101011000101 10

QoL011011001011 10101 1010110100100011 1000111101000

91011010000100110110100100111000190‌1 1000010111000

4100111 10111001001 0000011c0ca010111011000001 110001,

OLdorrrsttoM191 1111 v0101001 010001 tt LuLoULOOdLOL LTT

4?01001010071111101001010C1011001100011100101001‌0

40000100001"010 100010001016 00011 11e02 101001001100

4610001 10101100101 1011010001011 1010101001001100111

911100110111 1000101000102 1ecoLL0001010111 100011101

§OLo1o01111000010101111101C 1001101111001 111060000

oot 1co1101 11110110011 0000GC11c0011 000000001001001

1111001010010010101001101‌0¢110¢1001 10110010001 1011

2i2010Lco10F1 1000001u0001C1 111011001 000000tIT1LOL0

90120100101111110011111001‌0C10011100001 10111101000

911106010000011 1011 01001001C01 10011 1101001011111 10

90111011010n011 10010011 0000cLu1010101111 100001 1001

1o101c0100171100010111‌11¢11 1000100110001 100010000

OLortiodiorerioriiccoocrarcoortoro1 Loro1111t10010

911111111010000100011‌01 10010001 1000101011110100‌10

Horst tororitr110010010000011 1100101 c01011 LLoLo1OLO

4811011101 9010000111100111011101010‌01 to0110011 10

011111010001001 1010001011111 10G001COL0Gr1O1111010

tooucoo1111111111 100010111011 1101001101 100011111

14211001} 1911120010111 1001c0011110010000011 101000

£1011111110100010010010010061111000101100100 1101 00

9111001 L091 00000 10000001C000110011 001 OoLOr

9v01101 1011 o010¢1001 1 1egoe1cL01 111 0000100101100011

41110100011 101111¢:01000001¢1000000c1001 1111100011

£61100011190101 1101 090001001000 1o00c000001 010101 11

1120011 Lor 101101001101 co11C11C01216110001 10010100

SCLoLo001 LonoLL NIA 1011 LooLLCTOOLOLOLOOIl1OL1110001

{LL LoL1101%0001010110010101001110111110110 1011100

1101000110110000001 11c00101c000011101111 1001000010

911100411 100001011 LoL L001 1011101001 10111101101 10

LLL 1010011 101010¢ 10110101 100001 100111 LOOLLOLIIEL

111110111011000000001 101061001 1101010000001 101100

1201011 1010¥010010111 11001 1cooL01011 10110011101 100

Le0L1C10110°000 100101110101 1101011011111011000101

41000100000011111021111000C1100010101000001 101101

9111110011090000100000010111 11 100000 101001100111 Lt

92010111000110111001100100011 11 1010011100001000000

901000000101 10100011 0001010110111111001 1001101101

GoLooooitzoM1001111110111 11C001001111 100001001100

90101101111 101011100011 1000110111001 10110011111011

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A table of uniformly distributed binary random numbera (cont.)

01111110110" 110001 0010101 Loco 1oL0010100001011 1011

9101160001191 oor tocoII 11 1ocoLLATtITOVOLOLLOLAAIOL

9110100101 1901 10110001 1e00¢¢11111001101010000000100

£11010010010111c010110010¢11101 00006110101 11000111

900000111007 10100011100100010111101 100011100101 100

0001111111 199100101 1oveGo00couL 100011111 t1oLOLOL Lo

1110011000101 1011101001 1001610001 1901 LooLOOOI L1LOL

2011110100101110111001G011¢L0010111021 00100101011

011000101 10700001101 1001010100001 00011110000010001

11L2120L011111100001111C101 101110101 1001110021 Lol

90001 100000"110100001001110c01110100011 11010110101

140101111101111010110001111110100110000100010011100

81001111100011111001000001 1coo00111111110100010001,

9110011101 1010011100111 100010u00001 1901 101101001 10

10000G0001011111001010110011100110010111110100101O

11011010011°01001001011010O¢01 101 100101011110101 10

810000001011111010110O1 1001111100011 101110001 LOL

910111111 101000110000110110100111 100001 101000001 11

1101100011 10000101 001010111 100000100101100O1 110011

000111010019001101111001100100001 10001 000001001001

9000010011171 10001001 190100¢1 100¢00001101010101011

811100100111000011001001100¢11001010010000011 000100

9111111001 101111100100010001011000000000101 1010000

9011000110101 1001 1001 106010c1 00010101101 100001110

ALLOLL 1901 Tt LCL0111 110011101011 1011101 100000010

orttorod1iieoLort11011 101 1011161001 1110001 10010110

Se0oo00011011111011110100101 1010010101111 101000001,

9100011001070110010100111111001400000011110010101011

11000010110%0110001101100100¢0001100¢10010010100000

110111111 10°101011000001001¢110000111110001110011 0

910000100001 100111000u91000¢111000010010111010100 0

0101 1000100101 110000101 101.c0L01010011 100010011001

901100161111 1001 1001101110011010110 1 1100101000001

900010001 1090010001 11001100¢0010001111111101 010011

1010000001101111101 10110001611 100111010001 10011100

111100010001011100001111001C011 10010111111 LOOLLL11

900100001 190100100101 101111c01111111110011001 10001

01114C0101191 100111001010001111111101 1001119010000

9010010110111100111010111111¢010012000101001111001 1

911100011019100000 0v0001010¢1 10101 Le010111111 10101

110101010001001110100100111¢01010001101001 10010110

01111011011 1000001111111001c001111100111010100010 1

9110001 10001111 101101000101 101.000110100010001010 01

1011100011110101000011000001111011100101 0010100100

901101100007001001010101011100110101110111001000 10

---Page Break---

A table of uniformly distribut sd binary random numbers (cont .)

4601261 12007101101 101110111 101010010110010000001001

40L00001101 11011101 g0010101¢01 100110100001 Lodbt Tt

$011100111010110011111101010000001c11 1010010001 Tt

{OLollooоor 111110106111 Lo01cI 111001 1ILt1O0011 LO0LT

4011000110000101 0000001110111 1co10011 1010001 ITT Ltt

2011001020% 1001101 00001000161 0011010111 000000001 10

9010111101101 1010001001 1101 10101000c00000000101101

91001101101 90101110¢101 101 1c1 G00n0NCOn001 11002 100L

92000000100 o1001co0100¢111 1001 coodd0c001001 100100

900011111007 110co0101 0011110011011 1001 LoLutogeeS

9000000001 1m1 0001001101001 roodu1000111100L000I rt

dot 1010110711111 10000001 1010011111 101001 0001001

0900101 100010001 100u11LoLoccouoItl1OOLOL TOLL Tl LOLe

9001010001191 10110100100001¢01000100010001 1LoLooLe

}o0010011001 100101110001 1cLootoorcoottILit1oL 104

9841001000090101001110011 100010100111 11010101 Lot

1901 L00L01101011¢101001111 1000011011110

9011001111 101010001 0010cov0101 10000101111 LoLtTOS

$t00001010110011000001111011010101 100101011 111110

G1L0LoLo11 A io111LL111 11 LOCOC1 12002110101 1010100100

91011101001°01000001101 1010101C1 10011111111 1000105

1010000000021110010001010111101 1101101001101 100108

0111000111 1P0000001 011001 101000011101 10110000 FORD

Patdtootttt torr roocito1 i112 LooLorciooLtt 1101000

tet011011001000010001 10001 tcLecor tL 1¢t1110011001000

102 11111000110011011010C00161111111102110000001 1100.

Jeooto011r1eL0011001 1010000c01010000000001110011LITT

4001 100001 0¢011 01000101 1c11 1001 uL01C 10000101 1O1OLL

QooLocoo0111 LLooF O11 1 LLoLcLoLLooOICOItI LOLA LALOE

9200010010110) 10101 10001111C000010010001100011000 0

§t200111 1000900100: oo1ci ci} FOIL LOOLoOLOL Laeloos

400111100; 1000110 doLoct ic1104 10011010011 Lobb

11110110011 10001001 110010101111 1010co1t toot LOON EE

G422001 1111900011101111C010c011001001 10001 00001014

901101101011011001011000 1 1010110001 1000010000105 ¢6

O21 10111001 1o10com10001 101011100101 190 IIT LoL tLOLS

oLoLoo1110J01001 100111 1o0oCLL2 1O10C1 LOLOL LOLLOLe

200100 L0d0P01011001111C101 100c001 1001 GOL OBOL LITO

9011111110001 tocooo01 c0cooocoLtoooLoOLIt LOLI AAtthS

140111101 100000011 0c010¢101C00000101 00011001 000010

$001010¢0011010100010110 11¢10001011¢01000000011100

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A table of uniformly distributed binary random nunbers (cont.)

O10LsGLiLOL8L11100111000111¢11610110001 10001 100110

O1GLAATLOLOOLLLL0L1111011010100111110100111 100111

9611101101 101110L01011000c01110110C10100000110100

901 Loiocovi ti 11tLociui0CL011011101011000000 1 00100

1?600010010111001001000000 11¢01101 1001010111011 1001

4600011111011 4001000001 1100¢0100101 101211011 110001

000001101001110011001011 00¢1000100001 10111011 10101

410110100111011000011011111100100010011011 00011100

9000001101711 101001110161 1Couco1 10011 100000001001

900000011101 10101010001 1100¢01 101 00¢10001111 000010

9001060001 19101000101000111¢r0011201111011 01001101

90101¢000119101010000010011111110101010011101 10001,

?901010100007010201111110111011 11 100010101101001 00

1010100000010001 100c001111610100111001112010001110

90001¢00010111001111100011 0¢001010001 1011000001010

001001 10100111111110110111110101111010001111 010010

90010011011000111100100101 0¢1011101 0001001111 00010

(9010001 101001100001 10011110¢100010101101110000 0100

200142010119 1001101 1101. 0000cLo1 10111 1010011010110

91010C000111101Lo0L1101C0111101 1001011101011 101101

100010L0100010101001101011c010111000111 1101000011

1101001101001010110010010 11C1001101011110010110000

9600101c00001001011 000001 10¢01 1000019000110000 1000

1210110t0111100001010011002¢0101101¢101001 001 L10t1

111010110191 000100011 10001coLo1titOOLI 11111100110

{NL LoLoLoLo1011111111211C0010111111110100010100

10111000019%001011 101010011101 10000001110010001001,

101011000107101011 102000001001 1010¢01 101100000100

9601140100071 1011101001 1coo1oraarorciii 10101100010

90010610101 90010110111101 110121 101101111 1000111001

9000101 10107000101 001010101 1010001011101 1010001111

1111110001 190000111 10101016¢1100101600001100100001

!10000011010100000101010011111111011 0000111101101,

90100¢001001 1010001 11001101¢1021100010100000111011

9010110010111 100101 10111101 10001101001000002 101011

119000001000010011011100100110001100100221011111100

0111001100001111011110 1C0101100000L0000111 10101111

OoLuttL11011 10001001211 11011100001 1000011 10011101

OLLor11L00101100L1010000101111100000101 10011111111

9011011100071100%000110010011101001101000100001001

©11000111001011000010010001¢1000111101001011 110001

(90010101110001011010011010 1¢01010000100010110011 00

0411000100100100111112011 10C0121201101 101000010110

11001101101001010001111010111010101 01 110110011111

9011101001011 100000010010101110111001110110 2001100

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A table of unitormly aisteiboted binary random numbers (cont .)

11 Locg¢00t tat torM91o11e0¢eco0111 LLeLLoLoLLoLLOOLl

OLLororitirerioriireorzco90101 1090010101 0011001001

MoOLocLo1l I 400101001 LoLo01couI Ll LIC rOOO LOLOL OTT 'ttovi0110vovo10L011We0L01courcoLOL Lori LIT toLort

41101 co0000+0001111110¢1eccorcorortttoLorioroolt

§ooL0CLOLor011001 10110111101 L11000101010111001 10

ieo1v101090%010111 001 10¢01 Cort 10100001 001 LooLOL OO

L011 LeLo001 1001100011 1or00cco1 l 1ocogoLoOLILILolt tT

§doo1cocortr1101111.G0101 101011 1010010000101 LOL

9101119010011 in1010196eecact 1000011101001 0000000

O11 1LoooLogeIOLIILOCI1011 16¢ 1 couIO101100101 LOLOL

900100110001 tod010110100c1 CCL 1611111200001 1011 1O0L

ALI LCOLoLIeLL0c001 1010101 1CLO1 101161010101 1010010

99oLoL0CoG1M10110001C0010C¢1011 00100100011 10110110

8101110011101100010001 0001 101011 10C1001 1001 0001 10

A ovoLdo1v1 101101 101001111 0¢101 11 00000L 01100111010

1100100000711 1 1010110001111 101100011001001 060000

LooLoL0 00rd torr LCLOLeLL E111 101110011 11000.00101

9011111101011 100100101 ci06co1 10111¢01110011 160001

02111000101 000011011 16100C1oucL 1011001001 110111 lt

91114696000¢00010001101C00G10u111 1010111100011 1100

901 LLG10000r 100011101 1aC010101cCc0001901010001 00110

10110111001 10101 100Cco10111101 Loroocortottooo1 ELT

vor1o1oo11t O10 001 1co1001cL0oLoLoot11 10101 101000

910u1111001 "1orer LocoLardoLcuLorooooooOI IIL ILLOLt TL

OoltLeotLosproorrtrtootootitirorttttLooLLoLedt tt

G22 11611101101 1 Loo9CLo01L01C1 18101610111000110100

40001 100101111 100101000101 e1010011 101 1001110000008

910L0va0000 ov0101101 voa1 1011061101 1000101 LoveLoLL

9000010110001 10110001 10111.6C001 10016011011 101 11OLL

QLO011L00101 10019000111 LoL LCLLGG1I001111 1010101101

1001111000 LOOLOLL1UL0G0LCIO0IIII1G00N1LOLLLLOL LE

ui 1LoG100°U1 10010001101 101101111001 1001101111100

91101C010001000G01 10011101 tcLo0L00001101 LoLodotoOL

11100101000}00101111111G00¢10100100000210100110014

S00 100C1001 11610011001 1ecocotoro1011111 1010000010

9000101 ovveorti901111111tCiOUI 1101010001 100111100

11011111 1001011610111 0010¢c1211 1000011 101001 O00

Aoci1o111oro9tooroco10111cciooti 101i tio01 10011110

{rt Ltolrrovt1ti01 1011 10111CL0101 11661111 OOULOLL

HlOLOLoc111¢101 Lo000r 1011 11001 1011110010101 110000

{et0001 11001 tyod09101o01co0c111 100101 1000010101001

tertotto101910110100001 0110000000101 t 101 00001 oe

40001110101 1611 1000c0110010¢1111000c1 100101 LoLotot

AvLoLororomroo11001110101C111111110011011 01000110

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A table of uniformly distributed binary random numbers (cont.)

11011116111 10001 1014c01 1010¢o200011¢ 1010011111000

0100111010001 1016011000111110010110100110011000

14011110u000010111100111001¢000000010011011101 1001

1400009002 10100011 161001100101010001 1011010001010

90000¢1001011110010C00101011100111 0C01111100160000

161000001001011001 100100101¢901 110110101 101111101

{110111010101 1010100611011611010101 1000L001 110010

2011111 100291 00co001101611111 0101 1011110001 1000101

0111111 1000010110101 1 1011101111101 110101 10001 10

91000000101 100011101111 11c011 001011 10101000100100

LOLI EET oLooL0L010111C11101011110011101 0000011

1011111 1100011101010100u00¢1001 1900001110000111100

9011101 10007001001 101100001C1110010100111011 100110

91010111110100100101101¢010¢100010011 1000000110111

1101001.0000r0011100000001001010000111 1000001010101,

91011010101 1010000000001111111010100011000101 00010

Oli L1or 11000011111 100111 10cL0011 111101101 11010000

11111100101°010110101 1001 0ccoogo1 10101011 LoLoo1 tt

{¥oooi1011191111100010000111010111111 101001000001

111111111 19000011 1011001001008110111111 01001 10010

oL1reLo01 1110101 14110012111 101200100011111101110

2olco100011111001001 10100611 101110C11001000111001

901 1011101011 10000111011110011000101010001000011

9101110101070011 10¢10101001co0C001100001001102 1000

900U011G001"111c001 100161 10¢1010001100100011101000

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91011010011 9101100101010¢L0100110C011001 1000.

39

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APPENDIX 2

A tablo of uniformly distributed decimal random nunbers

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A table of uniformly distributed decimal random numbers (Cont)

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