- PUERTO RICO NUCLEAR AND RANDOM NUMBERS FROM A RA 101 SOURCE OPERATED BY UNIVERSITY OF PUERTO RICO UNDER CONTRACT NO. AT 40-11-1829 FOR U.S. ATOMIC ENERGY COMMISSION ---Page Break--- Random Numbers from a Radioactive Source Arnoldo J. De Hoyos* and Donald S. Sassacer * Submitted to the University of Puerto Rico at Mayaguez, Mayaguez, Puerto Rico in partial fulfillment of the requirements for the degree of Master in Science (Nuclear Engineering) - Work Performed at the Puerto Rico Nuclear Center, Mayaguez, Puerto Rico. ---Page Break--- University of Puerto Rico College of Agriculture and Mechanical Arts Mayaguez, Puerto Rico Random Numbers from a Radioactive Source by Arnoldo J. De Hoyos A thesis submitted in partial fulfillment of the requirements for the degree of Master in Science (Nuclear Engineering) June, 1965 Members of the Graduate Committee Date

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______________________ ---Page Break--- ACKNOWLEDGEMENTS I wish to express my gratitude to the many persons who collaborated in this work but especially to Dr. Donald S. Sassacer for his advice, help, and continuous encouragement in the performing of the whole thesis. I also wish to thank all the personnel of the Computer Center of the College of Agricultural and Mechanical Arts and especially to Mrs. A. Kay and Mrs. A. I. de Alearin for their help in programming the conversion and test of the random numbers. This research was performed during the period when the writer held an International Atomic Energy Agency fellowship at the Puerto Rico Nuclear Center. ---Page Break--- Abstract The statistical fluctuations in the disintegration of a radioactive source measured by radiation counter laboratories, a multichannel analyzer and a gamma-ray scintillator detector were used to produce a sequence of uniformly distributed binary random digits. This binary sequence was then converted to a decimal sequence and seven tests of randomness and uniformity of the distribution were conducted.

appliea. contradict the hypotheses that: *) The distribution of the decimal digits fits the uniform distribution (frequency test) *) There exists no correlation of association among the digits of the sequence (auto-correlation, mean square difference and serial tests) ©) There exists no abnormal clustering or dispersion among the digits (runs tests) ©) There is not a favored five digits arrangement (poker test) ii ---Page Break--- ‘Tables of the uniformly distributed binary and decimal random digits are presented. ‘These random numbers or pseudo-random numbers generated from these may be used in statistical sampling and in Monte Carlo calculations. The method presented in this thesis to generate a sequence of random numbers compares favorably to other methods that are reported. ia ---Page Break--- TABLE OF CONTENTS ---Page Break--- el a Table saw LIS? OF TABLES ---Page

Break--- ergopuctros ‘The purpose of this work was to generate random numbers by using the statistical fluctuations in the disintegration of a radioactive source. A sequence of numbers is called random when there exists no dependence among its members. That is, each number gives null information in guessing other numbers. The Monte Carlo method consists of solving a physical or mathematical problem by using random numbers to simulate a random process that is directly or indirectly related to the original problem. Besides their use in the Monte Carlo method, random numbers are widely used in statistics as a safety rule against bias in sampling and also to give validity to tests performed in the design of experiments. Random numbers are generated by two general ways: arithmetical and physical procedures. Arithmetical procedures consist of iteration methods based upon a mathematical formula that generates pseudo-random numbers. These numbers are only random from the point of view of some specific application or by passing several statistical tests for randomness. Physical procedures consist of a physical device (e.g., dice, a roulette wheel, a radioactive source), used to generate a sequence of random numbers. ---Page Break--- REVIEW OF LITERATURE In 1949 J. von Neumann said "Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin" (1). However, up to now the most common way to generate random numbers was arithmetical: The congruence method. Why, if we are convinced that our deterministic mind cannot conceive ways to generate random numbers, do we insist on the arithmetical process? The answer in 1959 of the International Business Machine Corporation to this question is that "Past arithmetical procedures (congruence methods) do exist whose results, though of course not random, nevertheless do furnish a satisfactory substitute". In addition, they are opposed to the generation of random numbers by physical procedures because “Nature tends to be systematic, so the

construction and maintenance of a mechanical or electronic device - a perfect roulette wheel - is not at all cheap or easy for practical necessities" (2). The report of Nac Laren and Marsaglia in April 1964 (3) concerned with the suitability of the arithmetical processes ---Page Break--- of generating random numbers showed that the sequences of pseudo-random numbers generated by arithmetical processes could pass many tests for randomness and still be unsatisfactory when used in Monte Carlo calculations for some practical problems; they suggest finally that the most suitable sequence of random numbers is that obtained from a table of random numbers, which itself is generated by a physical process. An answer to the second argument of the IBM Corporation, which was the same as that of Brown from RAND Corporation in 1949 (4), concerning the difficulties of generating random numbers by using a roulette wheel is that a roulette wheel is not the only physical process that can be used to generate random numbers, in spite of the fact that RAND Corporation used it for the construction of a 1,000,000 random digits table (5). Perhaps the strongest argument against generating random numbers by physical processes is, as J. V. Neumann pointed out in 1949, the practical need for reproducibility, that is, of repeating the calculations exactly. Still, it seems that there exists physical phenomena where randomness is essential. Nature apparently is not completely deterministic. ---Page Break--- If the physical process is incorporated directly into a computer, as it generally is, then this criticism may be answered by noting that it is always possible to print out a given sequence of the computer input random numbers and use this sequence to repeat the calculations. Another possibility is to look for systematic errors in the actual running of the program by performing statistical analyses of the results. Kahn in his research memorandum of 1954 "Application of Monte Carlo" (6) claims that it is impractical to print a sequence of random numbers.

generated by a physical process because of the limited memory and input-output capacity of computer machines. However, the practicality of using a table of random numbers (generated by a physical process) for Monte Carlo calculations was demonstrated by MacLaren and Marsaglia in April 1964 (3). They used different sections of the computer memory alternately and as a buffer, and concluded that a table of random numbers is the most suitable way to generate a sequence of random numbers. Since it was found that in several cases pseudo-random numbers generated by arithmetical processes were unsatisfactory and also, since modern techniques make the use of random numbers generated by a physical process practical, it is now reasonable to start thinking again of better methods for generating random numbers by using physical processes to generate random numbers. Several investigators who used physical processes to obtain tables of random numbers are: Tippet, who used census reports (7), Kendall and Smith, who used a mechanical roulette wheel (8), Hamaker, who used a rolling 10-sided die (9), and the RAND Corporation, which produced the largest and most used table by means of an electronic roulette wheel (5). Research on the use of a radioactive source to generate random numbers was done by J. Von Hoerner, who generated a sequence of random binary digits by considering the position in time of a flip-flop activated by a radioactive source counter (10), and also by M. Isida and H. Takeda, who generated and incorporated into a computer a sequence of decimal random digits obtained from the last digit of a pre-set time radioactive source counter (11).

THEORETICAL CONSIDERATIONS

Experimental observations of a radioactive source show that the radioactive decay occurs at random and at independent moments of time. It is known that radioactive decay can be represented by a Poisson distribution (12) which tends to the normal distribution with increasing counting rate (13). In order to use radioactivity to

generate random numbers with some specific distribution it is convenient to convert the non-uniformly distributed random numbers obtained by counting a radioactive source, to a sequence of uniformly distributed random numbers. This is accomplished by comparing the successive number of counts of a radioactive source and assigning a one or a zero to the comparison depending upon certain criteria. By this process the original non-uniformly distributed random numbers are converted into a sequence of uniformly distributed random binary digits, which are then transformed into a sequence of uniformly distributed random decimal digits. The advantage of this method over the other physical methods mentioned earlier (10, 11) is that the final distribution of random numbers is not affected by the variance of the original distributions. If one is interested in incorporating the radioactive process directly with a digital computer, then a slight variation of this method gives faster and therefore better results. For example, the fact that in two successive disintegrations there is the same probability that one takes longer than the other can be used in connection with an electronic clock to feed a sequence of uniformly distributed random binary digits directly to a computer.

EXPERIMENTAL PROCEDURE Generation of the random numbers A Radiation Counting Laboratories' 512 multi-channel analyzer with a Tally tape perforator and a gamma ray scintillation detector were used to measure the activity of a Cs-137 source of approximately 1 microcurie. The analyzer was used as a preset time scaler in the automatic mode. The information in the channels (counts accumulated during 0.1 seconds) was readout in perforated tape. This tape was converted to 18M cards. Conversion of the original non-uniformly distributed random numbers to binary and decimal sequences of uniformly distributed random numbers. A sequence of uniformly distributed binary random digits.

was formed whose mth term was defined as 0 if Nem was less than Nong, or 1 if Neq was greater than Nemes, where Nzm and Neamt were successive channel counts. Each exclusive set of ten binary digits was then converted to three decimal digits, producing a sequence of uniformly distributed decimal digits. ‘See photograph next page. “The conversion was performed by an IBM 1401 computer. ---Page Break--- ‘Sequence output TeuTbT IO 9y3 99ez0UI6 03 pasn 3ueudENbs SequowoTduco pue zozkteue TouuRYO-T4 TMU TY aya Fo YdexBo30ya “1 eanbTa a ---Page Break--- ry Tests for randomness of the decimal digit In order to measure the independence of the decimal digits and the uniformity of their distribution, four blocks of 1000 decimal digits each were obtained and tested for randomness using lags 1 through 10. Lag 1 is defined as picking every consecutive number Ny, Ngs+++) Njggg of the sequence of decimal digits in a block as the random quantity, Lag 2 as picking every other number N,, Nq,-- for Nas Ngeeey Moo of the sequence and go on. Five of the seven tests used are reported in a very recent work of Carlsen (15). They are as follows: 1. The frequency test which consists of calculating chi-square $x = \Sigma (f_i - 100)^2 / 100$ where f_i is the frequency of the decimal digit i (i = 0, 1, ..., 9) in the block. The average and variance are also calculated, the obtained values are then compared to the expected values. 2. The serial test which consists of "binning" the numbers into a 10 x 10 matrix. A 1 is added in row i and column j, when digit i is followed by digit j (i, j = 0, 1, ..., 9). The expected result would be 10 in each position of the matrix. The following χ^2 is then calculated. $\chi^2 = \Sigma (f_{ij} - 20)^2 / 10$ i,j where χ^2 should be distributed as a chi-square with 90 df. The expression z = (2(χ² - x■))'/x■ - (2 x 90 - 1)'/2 is used as a normal deviate with unit variance and the observed value of Z is then compared to expected values. 3. The auto correlation

test which consists of calculating the auto correlation coefficient of Pa, sar ah for h= 0, 1, 2,..., 10 where ¥ is the mth term in the lock and comparing it to expected values. 4. The runs test number 1 which consists of finding the runs above and below the mean. To find the runs above and below the mean a sequence of binary digits is formed whose mth term is defined as 0 if M is less than 5 or 1 if M is greater than 4. A subsequence of k zeros (or ones) bracketed by ones (or zeros) at each end forms a run of length k. Runs are counted and compared to the expected values. ---Page Break--- 43 5. The mean square difference test which consists of calculating $\frac{1}{2000} \sum_{m=1}^{1000} (Ny - Nag)^2$ where d= 1, 2, 3, 4, 5, 10, 100, 101, and comparing it to the expected values. In addition to these tests, another runs test and a poker test recommended by Brown (16) were used. 6. The runs test number 2 which consists of finding the frequency of runs in the decimal sequence, and comparing these values to expected results. 7. The poker test which consists of scanning the decimal digits in groups of five digits each, simulating Poker hands, and finding the frequency of seven classes of hands; busts (symbol abcde), pairs (symbol aabed), two pairs (symbol aabbc), three digits alike (aaabc), full house (symbol aaabb), four digits alike (symbol aaaab) and all five digits alike (symbol aaaaa). Results are analyzed using a chi-square ---Page Break--- RESULTS The following results are based on a sample of 4000 decimal digits. Averages are referred to the four blocks of 1000 decimal digits each. ---Page Break--- wopeers Jo seer6eq = ac eg" peaoodxs, 629° peazosao, 0S" peaoodxa gtr peazaeqo a9.6 303 (°/, 56) 2X 30 ONTeA Z6*OT WeUR sseT poyoodxE $91°6 —_poaresao 00h Oh Oh OO% ODF OOF OOF COW COk OOH poRDedeE 66 GRE eth GLE 6th Ty TEC Low 2LE 2c veazesco sou yzeA pBeroay exenbe=FuD. Aouenboxg ¥ oto 9803 Aouenbora *t etaeg ---Page Break--- as oT OO EO TT OT BOT 6 fez"0. TS'26 -H2"tt SL*EOT 8 ‘Ls0°0 92°06 = h2"TT SO*TOT 2 ‘L10°0 92°68

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€e°6t oL°6t £6°6t ol+6r 4 6t ae" 6t 29° 6t 16°6r Su" 6r 99°6 46° 6t So-0z 90°02 Us6T OT'0z TT"07 66-6t Sa°6t dovoz 99°61 n6-6T 86°61 S6-6r S0°02 60°02 62-60 19°6t 66°61 26°6t 16°6t T1092 Se°6t 66°6r 19°61 96°6T 46° 6t £0°07 86°61 96°6T E002 to*oz 2ST 100s HOO 0°02 9T-02 26°61 26"6T 9t"O? OO" oz 68° 6t Sg°6t zo-oz 20°02 S6°6T 60'oz 40°02 9T°02 90°02 fnt'02 0°02 Sovoz 00°0z Lo*oz Er'oz 9°02 Lovz? 16°6t £or02 00°02 26°6t 98°6t 00°02 tO"Oz HOrOZ Te te0e 60°02 St*oz Soroz eto 0°02 z0-oz E02 46"6r Te"02 gor07 tr'0z Et-oz £6°6r g0°02 ot-02 Hore ESE ot-e2 92 9f 92 gf a2 9€'g2 otra2 992 st'92 ire) 266 96°6T 9E°g2 peazesqo or e I> 3893 uoTaeTexz09-o3ny a zy *€ erquy o =a ---Page Break--- OS OOS SEG ONL TST CIE OSI OO THT OO ORE ‘Sunz Jo TequNG paqeder OT 2" Bot ee Sed 2°oT SI eo IS "Ott Ge" Ohe 6 OSH —S2-6 00°9 COTLT SL'OE Sz2°T9 Sz'oeT 00° 0S 3 OS*ash «SLL S2°S Se-gt Sz-0F SL°99 G2"BIT 00° OK z Ge06h 006 -OS*S = SB*ZT OS*EE O0'RS GerdeT SL he 9 00°06 G0"g G26 GLr2t OS*EE 00'S OS-ozT 00°0S2 s OO°66H SL" 00°G = GL*HT OS*OL 0S*19 OS*tat 00° Se t SL°66h SL°2 -SL'2 SLT Se*92 SL°h9 OS'ozt 00'9S2 € SL*00S 0S"9 OSG Se"9T SL*zE OSD oOrzeT Ger ese z OS'00G 00°S SEL OS*ST OSedz Gerl9 Geet SL*tSz sunz Jo zequnu T OSHTS OO'g GL*L SLOT OS*TE O0°SL SuEet SL'TS2 eBezear poazesqo Ber hav =o $ + E z T ‘SuRE FO WBIOT eau 943 soteq pur enoge sun 13803 cure +) otqes ---Page Break--- BOR OST OST BOT ON oT aa ST tor oot W peqoodar OF SOSH LOH LET SLT SIN OT EO IST € — 6G6-hT EGEHT S69°9T 6ErOT HIZ-OT LEL OT zeHroT gSE-or G Shovet USS'ot BLELt GLH-OT SozOT Lys oT tOS'oT HEH oT 4 220°ST OLE'ST YOST HTO"ST GLL'OT LoS-oT onN'ot z29°oT 9 WE"HT OfL-tt LoT-9T EoK-ot E6S-9T zee-oT OOkST JgL-oF S

wBL'HT G9THT SloroT S6EoT AzeOT SHOT Gert gos or % — SBErkT O99"HT 9EE-OT naT“OT ODOT ZLHOT EthtOT THH OT € —ndorst Lge"St 2hErOT Of'oT GET 962"9r goL"OT H64OT 2 WT*ST OT9"HT OTT“OT 6ST'OT EBE-9T GhL-Gt COROT E69°9T u T — AOT'ST OB0'ST L6O'ST LS0-9t 6l2"9T G9H-OT TL9"ST HhL-gT peazosqo Ber tor 00T OT t € z T =P 3903 eouozez5yp orenbs wesw °S etary ---Page Break--- Table 6. Runs test © Length of run 2 3 5 tag Observed average 821.00 77.50 7.00 0.75 0.00 1 number of runs 626.50 76.25 5.25 1.00 0.85 2 804.75 80.50 9.50 1.50 0.00 803.75 79.25 10.25 1.00 0.50 800.00 86.00 8.70 0.50 0.00 810.25 79.75 8.50 1.25 0.00 810.25 80.00 9.25 0.25 0.25 7 863.75 75.75 6.25 1.25 0.25 8 £12.00 77.75 10.25 0.50 0.00 9 602.75 63.50 8.75 1.00 0.00 10 Expected number of runs 610.00 61.00 8.10 0.61 _0.09 ---Page Break--- EIS SATA CORT URNS eS, STEMS TS pays GOO OSE ORL OD*LG-OH'9G-—OR*EOH 26TH AouoNbaxs pexsadka OT Te o 9 o ic) 5 eee toe 6 wz 0 € s 4S sé oth tz 8 260° 0 * L 19 99 ath ke 1 o%fth 0 » 8 &s a 9 ue 0 * tt €9 el eh ah S 979 0 2 2 tL 61 20h SEZ h Great ot € or 19 19 oth lke € tore 0 z L 2b SL otth tee z wr 0 € L 0s 46 Soy tz2ouonbexz T re 0 z + 98 To OF © TG2—peazosqo Teeeve) (qeewe) (ageee) (Squaw) (oqqwe) {poaee) (Spoqe) —puey yo Ser 4X Senta sanog senoy {Ind soozys slyed Ons Tea 38M SECT 3893 soya *L oTquL ---Page Break--- ee DISCUSSION OF RESULTS Frequency test (table 1 An inspection of table 1 shows that all the digit frequencies are close to the expected value of 400 for a random sample of 4000 uniformly distributed decimal digits. Five of the digit frequencies are above and five below 400. The digit zero has the highest frequency 432, and the digit one has the lowest frequency 372. The even digits have a total frequency of 2017, and the odd digits have a total frequency of 1983. The chi-square for these digit frequencies has a value of x^2 = 9.165 which is smaller than 16.92, the critical value of the chi-square for 9 DF at the 95% confidence level. The blocks 1, 2, 3, 4 of 1000 digits each, give the respective values.

for the chi-squares of xf = 12.02, x72 = 10.86, x8 = 9.44, and x3 = 12.64. The average of these chi-squares is ¥7 = 11.24. All of the chi-squares have values that are less than 16.92. The average of the digits is found to be 4.48, which is somewhat lower than the expected value of 4.5 for a uniform distribution in the interval from 0 to 9. The averages of two of the blocks were below and two were above 4.5. ---Page Break--- It is interesting to note that the averages reported by Carlsen (15) of 100,000 uniformly distributed pseudo-random numbers obtained by congruence methods and Brown (16) of the 1,000,000 digits of the RAND table (5) obtained by using an electronic roulette wheel are both below the expected mean for the corresponding uniform distribution. The observed variance was 0, a value that is lower but close to the expected value of 0.833. Based on the law of large numbers, all of these results should improve with increasing size of the sample. These tests give no evidence to reject the hypothesis that the decimal sequence is uniformly distributed. Serial test (table) The object of the serial test is to indicate the tendency of given digits to be associated with any other digit. The values of z for the different lags are less than the value 1.96 of z for the 95% confidence level. This test indicates that the hypothesis of mutual association of digits is rejected. ---Page Break--- Auto-correlation test (table 3) The results from the auto-correlation test indicate that the average values of ch for different h (h = 0, 1, ..., 10) and different lags (lag 1, 2, ..., 10) are somewhat lower but nevertheless close to the respective expected values. It seems reasonable to assume that these values should be low due to the fact that the observed average value of the digits is lower than the expected value, as is shown in the frequency test. This shows that there exists no significant evidence of correlation among the digits. Runs test 1 (table 4) Runs above and below the mean give a measure of the tendencies of

the digits to cluster or disperse with respect to the mean. The observation of the values presented in table 4 clearly shows that there exists no significant evidence of abnormal clustering or dispersions of the digits with respect to the mean. “Notice that correlation is not a measure of independence but only of linear dependence (13). ---Page Break--- Mean square dissimilarity test (table 5) The mean square difference test provides a measure of the association among the digits. The average values found of M for the different @ (a © 1 2, 3, 4, 5, 10, 100, 101) and different lags (lag 1, 10) are also lower but close to the respective expected values. As with the correlation test, the observed values of M should be low since the average of the digits is less than the expected. Therefore there exists no significant evidence of association among the digits. Runs test 2 (table 6) The consideration of table 6 gives an indication of the degree of clustering or dispersion of the digits among themselves and shows that this type of anomaly is unlikely to be present in the sequence Poker test (table 7) Table 7 gives an analysis of favored arrangements of five digits, showing that the values of the frequencies for the different hands (busts, pairs, two pairs,..., fives) and different lags (1, 2,..., 10) except for lag 1 are close to the respective expected values. ---Page Break--- Fitness to the expected results is tested with a chi-square, and it is shown that all the values of the chi-squares except that for lag 4 are less than the critical value of 12.59 for the chi-square of 6 DF at the 5% confidence level. The anomaly which occurred in lag 4 is easily explained by noticing that this lag has one very unlikely but possible arrangement of fives (aasaa). The presence of this arrangement gives the main contribution to the large chi-square obtained. Advantages and disadvantages of the method used in this thesis to generate random numbers The arithmetical procedures have the advantages that the numbers are easy to

generate and easy to reproduce, but have the disadvantage that they cannot generate random numbers. Pseudo-random numbers generally pass various tests for randomness, but this only proves that we do not know the type of dependence established in their generation, and therefore we are not able to test for it. Nevertheless it is true that in some problems some types of correlation may not affect the results and in those cases arithmetical procedures may be more desirable. The methods using a radioactive source to generate random numbers have the advantage over other physical methods that the necessary equipment is easy and cheap to maintain and that mechanical devices which give special tendencies in the sequence of random numbers are eliminated. As mentioned earlier, the method used in this thesis when compared with the two other methods that use a radioactive source to generate random numbers has the advantage that it does not depend on the variance of the counting rate. M. Isida and H. Ikeda, for example, who used the last digit of a count to generate a sequence of decimal digits must keep the average counting rate (variance) greater than 2 in order to have a relative error in the uniformity of the distribution less than a, where a is the maximum difference of frequencies among digits. S. von Hoerner used the false assumption that the average is in the middle between odd and even numbers. The error in the uniformity of the distribution is inversely related to the variance. Perhaps the strongest assumption made in the methods that use a radioactive source to generate random numbers is that the process of radioactive decay is stationary with respect to time, whereas the activity decreases with time by a factor of $\exp(-\lambda t)$. The factor of decrease $\exp(-\lambda t)$ for the method used in this work (Cs-137 source, 0.1 second counts) is neglible, $\exp(-\lambda t) = 0.9999999999$ for 1 year = 30 * 365 * 24 * 3600 seconds. This shows that the assumption that the process is stationary is a very good one.

approximation. ---Page Break--- CONCLUSION ‘The method used to generate random numbers in this work appears to be better than the other physical methods reported in the literature. The physical methods have the advantage over the arithmetical process in that they generate random numbers. ‘The sequence of uniformly distributed decimal random numbers successfully satisfied all of the seven tests for randomness performed. Tables of uniformly distributed binary and decimal random numbers are presented in appendix 1 and 2 and are available also as index cards that can be used to draw random samples in statistics and to generate sequences of Pseudo-random numbers for Monte Carlo calculations as suggested by Metropolis and Marsaglia (3). These results suggest to one who wishes to make use of the Monte Carlo method that the construction of a device to generate random numbers using the disintegration of a radioactive source might be incorporated directly into a computer with advantages. ---Page Break--- Q) (2) a) (9) (20) 30

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Corporation P-4% ---Page Break--- APPENDIX 1 A table of uniformly distributed binary random numbers 9121001 111 000G010:001 v20000110100000111 10011110001 910101100001 10101001101 0001100000110110010 1 101011 Lit oo11 1002110011 100110100111 1001 101001000100010 1110000100111001101 1000101 co010011oL01 00101 Lou 91100110110000011111010000co010100001 101 100001001 G9OLOL0OL010L00111112201 02 1100121110111 1oo0LOLOrT 20101101110101710000110000111110000110010000 1 10010 9011110101001111010001101 10¢1 1000210010011 101008 24001101100110011110111110010101111100010101 101001 90111111010910100011011010112001110011101 1 11000101 1011001 101100001000100101101100101101000001 10C0001 1000100110010010000111101111110010101001 01 00100011 000001110001 00010001001 1141c11121001001 L1111 000010 9110111110100000110101100 01 1010000111101 11000119 2101 1001010700100010011001 1¢00010010101100101 10100 911101100000010001000101101¢10101100001 00000101001 4100001100101 211101120011 0001110110100100111 10081 90011100101901001101110000 1coL000101001111101 01111 9021110001 100000000011L0101CL011110111 0001 01011101 91110111010100001001 0001 00¢10101111101 11100100100

4€0010100000011100100100001¢11101110010100010100 00 100100000111111011110111010100010000101 00100001111 922011001101000100011111010111012 0100001011111 1110 99010000000" 10101101011 0000c1000011000100000001111 1000100010100100001000 1011¢10001111001011010100000 8101000100101011011110101011120101001 11010100001 19 91111011100900100 1 100111011C011001100001011 1001109 94410010100°000111 ic101010¢101011101100101011 10110 120100100111011011100001010¢11 000101000101111110 10 200010111010100101111101100110100101001010010100 10 911001110100010030010101001 110000001011 1011110000 31010C001 1001010101 10901110101 10011 50001 tL LOD ToF 991111111101 101c00010100100c0L000011101 10110100010 0110010000001 10110011100110100000001010101 01 100100 12010011101011200000011111 10201 100000111101 L01110 40011111110120111110100011 1¢10010111001101 00010111 1011100100011 10001 :o000010111101100001 100011 1001 10 901111010001011001 101 0000001201 1010001 100011011101 gtoL010101 0101 1101000001 10¢11001000011000010011 00 92010101100710010101000101001041101101000 1 1100101 | 110010101011000011 100106001 100100001 101111010011 10 4211101010101111100101002 11c00000101 1000110111 1010 9100101 1010100001100110000 0C100111 11010101001 10101 40000101011011000101001110 0C11210011101000000000 11 11001190001101100010001 1010101 1010010010111111111 10 ---Page Break--- % table of uniformly dictributed binary random numbers (cont) tettocoio01) 11001107111 1101100001101 1601 0100010010 1011001101703 1011010010101 Loot Lortcionootol tiitt {11111 LoL01¢00010101 1100010119111 00¢000001 10001010 }eootototvicios 1111011 161G0100011 101 111101100101 01 Go00oco0001 900101004111 1000C1 1010101 10010111 101101 4011 12CO01O0110111011101111220101 1001 10000001011 920101000 1070090111 14900001 111C10000001 00001101000 90100011100111011 00c000100010101111111 1 10001100000 $00000000111110011100011100100021111100100101 10011, 0140011110071001 1001101111 0¢1 00LoL OC LOLUNL TOON LTT Q1110011001) 1o101010010¢c01C1 v0100101000011 1100101 1001C0011190101110100110 11 104400101001

10000011000 9100110101 0011011011100100lc1 100001 0100000111101 00 Yorrocoacorriootttt iii itoIcLAtLoLolttorolLol ioe 41100111000 100100L101112111C01111 LocoLoL tLOOLLtT ET $eLLo1oL010010101 110001111001 1101000201011 111000 {4000t10000701111111111001lco101100101101 100111001 990010100101 110010001 101010101 00001 10010111 1000001 2011000101 1M101C0101010G001110010L0C001L LI LALO IL 9010001 110111110011 101 0cco0coLcL001111 101101010000 1900160C00170100111¢111010010010101110ll 1010010000 d40104011072010101ll110C001¢C100100010011 1011000100 90010110010101001011001111111000010001O1 0000001009 401111 1000111100000100c000c00101 000000001 10000000 190011011 109000111101 1010111101001 1000000000101 tt 00040014000 1000101 00001 1actu10011011110201 000001 9041100101191 10104 00110101 100100011 10101 101001110 {90L010111001110000100l 00001 101 1oUL0L1 10101011011 9000101 1011101 LooLo11010L00210101001 11 LouLOLOL TOO 011 OLorL0FOL 10110000111 20C1200111 1001111 10000111 {oLoct 101s ro1t011 100001 101121111111 1001100110010 41110¢001001111001000u100111111010001111 0000001001 SoLorcoLtL 1 YOOL10111011 1000121020001 LoL Lt tool LAI tt 30010100001"01101 100001 1010C01111001 0011010010011 901111011101 1000100101 00010ct0011 1000too101 1010111 Botot140111"01210111110111110000100100ll001 1010100 9101101 100000011101 1110011010101111101110001111010 $11 40100101711100010010101OC10111001 11000000001 111 $6101010000100011011101000lC01 10011 LonLo1L LooLorot 901001011111001 10101001 L001¢001 0001112100001 111104 91001100111000100011110010400000100001100010010OO 9100001 1100110010001 101001Coo0t 10010101 1101110010 9odd001011 19011 100132 1001 11C1L111110001 LoLt LocooOL $10101101019010110000100100¢2110100010110110101000 100101111001011101011011001 1001010011001 1011001111 ---Page Break--- A table of unifcrnly aiotesbuced binary random nusbere (cont.) !10000000101 10% : 100001 10001111 100110010011 201001, 0101111010111091 1101001 ¢o01co0c1 101 1101011000101 10 QoL011011001011 10101 1010110100100011 1000111101000

910110100001001101101001001110001901 1000010111000 4100111 10111001001 0000011c0ca010111011000001 110001, OLdorrrsttoM191 1111 v0101001 010001 tt LuLoULOOdLOL LTT 4€010010100711111101001010C101100110001110010l0010 40000100001"010 100010001016 00011 11e02 101001001100 4610001 10101100101 1011010001011 101010100100110011l 911100110111 1000101000102 1ecoLL0001010111 100011101 §OLo1o01111000010101111101C 1001101111001 111060000 oot 1co1101 11110110011 0000GC11c0011 000000001001001 111100101001001010100110l0¢110¢1001 10110010001 1011 2i2010Lco10F1 1000001u0001C1 111011001 000000tIT1LOL0 9012010010111111001111100l0C10011100001 10111101000 911106010000011 1011 01001001C01 10011 1101001011111 10 90111011010n011 10010011 0000cLu1010101111 100001 1001 1o101c0100171100010111111¢11 1000100110001 100010000 OLortiodioreriori iccoocrarcoortoro1 Loro1111t10010 91111111101000010001101 10010001 100010101111010010 Horst tororitr110010010000011 1100101 c01011 LLoLo1OLO 4811011101 9010000111100111101110101001 to0110011 10 011111010001001 1010001011111 10G001COL0Gr1O1111010 tooucoo1111111111 100010111011 1101001101 100011111 14211001} 1911120010111 1001c0011110010000011 101000 £1011111111010001001001001006111100010110010011O1 00 9111001 L091 00000 10000001C000110011 001 OoLOr 9v01101 1011 o010¢1001 1 1egoe1cL01 111 0000100101100011 41110100011 101111¢:01000001¢1000000c1001 1111100011

£61100011190101 1101 090001001000 1o00c000001 010101 11 1120011 Lor 101101001101 co11C11C01216110001 10010100 SCLoLo001 LonoLL NIA 1011 LooLLCTOOLOLOLOOII1OL1110001 {LL LoL1101%0001010110010101001110111110110101110O 110100011011000000111c00101c000011101111 1001000010 911100411 100001011 LoL L001 1011101001 10111101101 10 LLL 1010011 101010¢ 10110101 100001 100111 LOOLLOLIIEL 11111011101100000000001 101061001 1101010000001 101100 1201011 1010¥010010111 11001 1cooL01011 10110011101 100 Le0L1C10110°000 100101110101 11010110111111011000101 4100010000001111110211110O0C1100010101000001 101101 911111001109000010000001011111 100000 101001100111 Lt

9201011100011011100110010001111 1010011100001000000 901000000101 10100011 00010101101111111001 1001101101 GoLooooitzoM100111110111111C001001111 100001001100 90101101111 101011100011 1000110111001 10110011111011 ---Page Break--- A table of uniformly distributed binary random numbers (cont.) 01111110110" 110001 0010101 Loco 1oL0010100001011 1011 9101160001191 oor tocoII 11 1ocoLLATtITOVOLOLLOLAAIOL 9110100101 1901 10110001 1e00¢¢11111001101010000000100 £11010010010111c010110010¢11101 00006110101 11000111 900000111007 10100011100100010111101 100011100101 100 0001111111 199100101 1oveGo00couL 100011111 t1oLOLOL Lo 1110011000101 1011101001 1001610001 1901 LooLOOOI L1LOL 20111101001011110111001G011¢L0010111021 00100101011 011000101 10700001101 1001010100001 00011110000010001 11L2120L011111100001111C101 101110101 1001110021 Lol 90001 100000"110100001001110c01110100011 11010110101 14010111110111101011000111111010011000010001001100 81001111100011111001000001 1coo00111111110100010001, 9110011101 1010011100111 100010u00001 1901 101101001 10 10000G00010111110010101100111001100101111101001010 11011010011°010010010110100¢01 101 100101011110101 10 81000000101111101011001 1001111100011 101110001 LOL 910111111 101000110000110110100111 100001 101000001 11 1101100011 10000101 001010111 10000010010110001 110011 000111010019001101111001100100001 10001 000001001001 9000010011171 10001001 190100¢1 100¢00001101010101011 81110010011100001100100110¢11001010010000011 000100 9111111001 1011111001000100010110000000000101 1010000 9011000110101 1001 1001 106010c1 00010101101 100001110 ALLOLL 1901 Tt LCL0111 110011101011 1011101 100000010 orttorod1iieoLort11011 101 1011161001 1110001 10010110 Se0oo0001101111011110100101 1010010101111 101000001, 91000110010701100101001111110014000000111100101011 11000010110%011000110110010¢0001100¢10010010100000 110111111 10°101011000001001¢1100001111100011100110 910000100001 100111000u91000¢1110000100101110101000 0101 1000100101 110000101 101.c0L01010011 100010011001 901100161111 1001 10011011100110101101

1100101000001 900010001 1090010001 11001100¢0010001111111101 010011 1010000001101111101 10110001611 100111010001 10011100 111100010001011100001111001C011 10010111111 LOOLLL11 900100001 190100100101 101111c01111111110011001 10001 01114C0101191 100111001010001111111101 1001119010000 901001011011110011101011111¢0100120001010011110011 9111000110191000000v0001010¢1 10101 Le010111111 10101 110101010001001110100100111¢01010001101001 10010110 01111011011

1000001111111001c0011111001110101000101 9110001 10001111 101101000101 101.0001101000100010100l 10111000111101010000110000011110111001 01 0010100100 90110110000700100101010101110011010111011100100010 ---Page Break--- A table of uniformly distributed binary random numbers (cont .) 4601261 12007101101 101110111 1010100101100100000l001 40L00001101 11011101 g0010101¢01 100110100001 Lodbt Tt $01110011101011001111101010000001c11 1010010001 Tt {OLolloooor 111110106111 Lo01cI 111001 1ILt1O0011 LO0LT 4011000110000101 0000001110111 1co10011 1010001 ITT Ltt 2011001020% 1001101 00001000161 0011010111 000000001 10 9010111101101 1010001001 1101 10101000c00000000101101 91001101101 90101110¢101 101 1c1 G00n0NCOn001 11002 100L 92000000100 o1001co0100¢111 1001 coodd0c001001 100100 900011111007 110co0101 0011110011011 1001 LoLutogeeS 9000000001 1m1 0001001101001 roodu1000111100L000I rt dot 1010110711111 10000001 1010011111 101001 0001001 0900101 100010001 100u11LoLoccouoItI1OOLOL TOLL TI LOLe 9001010001191 10110100100001¢01000100010001 1LoLooLe }o0010011001 100101110001 1cLootoorcoottILit1oL 104 98410010000901010011100 11 100010100111 11010101 Lot 1901 L00L01101011¢101001111 1000011011110 9011001111 101010001 0010cov0101 10000101111 LoLtTOS $t0000101011001100000111110110101 01 100101011 111110 G1L0LoLo11 A io111LL111 11 LOCOC1 12002110101 1010100100 91011101001°01000001101 1010101C1 10011111111 1000105 10100000000211100100010101 11101 1101101001101 100108 0111000111 1P0000001 011001 101000011101 10110000 FORD Patdtootttt torr roocito1 i112 LooLorciooLtt 1101000 tet011011001000010001 10001

tcLecor tL 1¢t1110011001000 102 11111000110011011010C0016111111110211000001 1100. Jeooto011r1eL0011001 1010000c010100000001110011LITT 4001 100001 0¢011 01000101 1c11 1001 uL01C 10000101 1O1OLL QooLocoo0111 LLooF O11 1 LLoLcLoLLooOICOltI LOLA LALOE 9200010010110) 10101 10001111C000010010001100011000 0 §t200111 1000900100: oo1ci ci} FOIL LOOLoOLOL Laeloos 400111100; 1000110 doLoct ic1104 10011010011 Lobb 11110110011 10001001 110010101111 1010co1t toot LOON EE G422001 1111900011101111C010c011001001 10001 00001014 9011011010110110010110001 1010110001 1000010000105 ¢6 O21 10111001 1o10com10001 101011100101 190 IIT LoL tLOLS oLoLoo1110J01001 100111 1o0oCLL2 1O10C1 LOLOL LOLLOLe 200100 L0d0P01011001111C101 100c001 1001 GOL OBOL LITO 9011111110001 tocooo01 c0cooocoLtoooLoOLIt LOLI AAtthS 140111101 100000011 0c010¢101C00000101 00011001 000010 $001010¢0011010100010110 11¢10001011¢01000000011100 ---Page Break--- A table of uniformly distributed binary random nunbers (cont.) O10LsGLiLOL8L111001110001 11¢11610110001 10001 100110 O1GLAATLOLOOLLLL0L1111011010100111110100111 100111 9611101101 101110L01011000c01110110C10100000110100 901 Loiocovi ti 11tLociui0CL01101110101100 00001 00100 1€60001001011100100100000011¢01101 1001010111011 1001 4600011111011 4001000001 1100¢0100101 101211011 110001 000001101001110011001011 00¢1000100001101 11011 10101 410110100111011000011011111100100010011011000111 00 9000001101711 101001110161 1Couco1 10011 100000001001 900000011101 10101010001 1100¢01 101 00¢10001111 000010 9001060001 19101000101000111¢r0011201111011010011 01 90101¢00011910101000001001111111010101001 1101 10001, ‘901010100007010201111110111011 11 100010101101001 00 1010100000010001 100c00111161010011100111201 0001110 90001¢0001011100111110001 10¢001010001 1011000001010 001001 101001111111011011111010111010001111 010010

90010011011000111100100101O¢1011101 0001001111 00010 (9010001 101001100001
10011110¢100010101101110000O100 200142010119 1001101 1101. 0000cLo1 10111
1010011010110 91010C000111101Lo0L1101C0111101 1001011101011 101101

100010L010001010100110101 1c010111000111 1101000011
110100110100101011001001011C1001101011110010110000 9600101c00001001011 000001
10¢01 100001900011000O1000 1210110t01111000010100110O2¢0101101¢101001 001 L10t1
111010110191 000100011 10001coLo1titOOLI 11111100110 {NL
LoLoLoLo1011111111211C0010111111110100010100 10111000019%001011 101010011101
10000001110010001001, 101011000107101011 102000001001 1010¢01 101100000100
9601140100071 1011101001 1coo1oraarorciii 10101100010 90010610101 90010110111101
110121 101101111 1000111001 9000101 10107000101 001010101 1010001011101 1010001111
1111110001 190000111 10101016¢1100101600O011001 00001
!10000011010100000101010011111111011 0000111101101, 90100¢001001 1010001
11001101¢1021100010100000111011 9010110010111 100101 10111101 1000110100100O002
101011 119000001000010011011100100110001100100221O1111100
0111001100001111011 1101C0101100000L0000111 10101111 OoLuttL11011 10001001211
11011100001 1000011 10011101 OLLor11L00101100L101000010111110000O101 10011111111
9011011100071100%00011001001110100110100O1 00001001 ©1100011100101100001001
0001¢1000111101001011 110001 (9001010111000101101001101O1¢01010000100010110O1100
041100010010010011111201110C0121201101 101000010110
110011011010010100011111010111010101O1 110110011111 9011101001011
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binary random numbers (cont .) 11 Locg¢00t tat torM91o11e0¢eco0111 LLeLLoLoLLoLLOOLI
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co0000+0001111110¢1eccorcorortttoLorioroolt §ooL0CLOLor011001 10110111101
L11000101010111001 10 ieo1v101090%010111 001 10¢01 Cort 10100001 001 LooLOL OO L011
LeLo001 1001100011 1or00cco1 I 1ocogoLoOLILILolt tT §doo1cocortr1101111.G0101 101011
1010010000101 LOL 9101119010011 in1010196eecact 1000011101001 0000000 O11
1LoooLogeIOLIILOCI1011 16¢ 1 couIO101100101 LOLOL 900100110001 tod010110100c1 CCL
1611111200001 1011 1O0L ALI

LCOLoLIeLL0c001 1010101 1CLO1 101161010101 1010010
99oLoL0CoG1M10110001C0010C¢1011 00100100011 10110110 8101110011110110001O001
0001 101011 10C1001 1001 0001 10 A ovoLdo1v1 101101 101001111 0¢101 11 00000L
01100111010 1100100000711 1 1010110001111 101100011001001 060000 LooLoL0 00rd torr
LCLOLeLL E111 101110011 11000.00101 9011111101011 100100101 ci06co1 10111¢01110011
160001 02111000101 000011011 16100C1oucL 1011001001 110111 lt
91114696000¢00010001101C00G10u111 1010111100011 1100 901 LLG10000r 100011101
1aC010101cCc0001901010001 00110 10110111001 10101 100Cco10111101 Loroocortottooo1
ELT vor1o1oo11t O10 001 1co1001cL0oLoLoot11 10101 101000 910u1111001 "1orer
LocoLardoLcuLorooooooOI IIL ILLOLt TL OoltLeotLosproorrtrtootootitirortttttLooLLoLedt tt G22
11611101101 1 Loo9CLo01L01C1 1810161011100O110100 40001 100101111 100101000101
e1010011 101 1001110000008 910L0va0000 ov0101101 voa1 1011061101 1000101 LoveLoLL
9000010110001 10110001 10111.6C001 10016011011 101 11OLL QLO011L00101 10019000111
LoL LCLLGG1I001111 1010101101 1001111000 LOOLOLL1UL0G0LCIO0IIII1G00N1LOLLLLOL
LE ui 1LoG100°U1 10010001101 101101111001 1001101111100 91101C010001000G01

10011101 tcLo0L00001101 LoLodotoOL
11100101000}00101111111G00¢1010010000021010011001 4 S00 100C1001 11610011001
1ecocotoro1011111 1010000010 9000101 ovveorti901111111tCiOUI 1101010001 100111100
11011111 1001011610111 0010¢c1211 1000011 101001 O00 Aoci1o111oro9tooroco10111ccioot i
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Lo000r 1011 11001 1011110010101 110000 {et0001 11001 tyod09101o01co0c111 100101
1000010101001 tertotto101910110100001 0110000000101 t 101 00001 oe 40001110101 1611
1000c0110010¢1111000c1 100101 LoLotot AvLoLororomroo11001110101C11111111 0011011
01000110 ---Page Break--- A table of uniformly distributed binary random numbers (cont.)
11011116111 10001 1014c01 1010¢o200011¢ 1010011111000 0100111010001
10160110001111100101101001100110000
14011110u000010111100111001¢000000010011011101 1001 1400009002 10100011

161001100101010001 1011010001010
90000¢1001011110010C001010111001110C01111100160000 161000001001011001
100100101¢901 110110101 101111101 {110111010101 1010100611011611010101 1000L001
110010 2011111 100291 00co0011016111110101 1011110001 1000101 0111111
1000010110101 1 1011101111101 110101 10001 10 91000000101 100011101111 11c011
001011 10101000100100 LOLI EET oLooL0L010111C1110101111001110100000 11 1011111
110001110101010 0u00¢1001 1900001110000111100 9011101 10007001001
101100001C1110010100111011 100110 91010111110100100101101¢010¢100010011
1000000110111 1101001.0000r0011100000001001010000111000001010101, 91011010101
1010000000001111111010100011000101000 10 Oli L1or 11000011111 100111 10cL0011
111101101 11010000 11111100101°010110101 1001 0ccoogo1 10101011 LoLoo1 tt
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1011001001008110111111 01001 10010 oL1reLo01 1110101 14110012111
1012001000111111 01110 2olco100011111001001 10100611 101110C11001000111001 901
1011101011 10000111011110011000101010001000011
910111010107001110¢10101001co0C001100001001102 1000 900U011G001"111c001 100161
10¢101000110010001110100 0 1101000101 0°0001101100011111 0101111 1001011 11000100
91011010011 9101100101010¢L0100110C011001 1000. 39 ---Page Break--- APPENDIX 2 A tablo of uniformly distributed decimal random nunbers 4630209
1852724136442571005434399923121 18205%690092
30406291796110316404467770826718785035556953687835
37050245244435098489822614997497361253103430662965
28995916929102627323591596244652539854290235 703000 8299180472068
36270604139038181 707324311 45897770839
43241512930086349477132546763806552114267761080577
98341751006442938605801554657618048 5672 32469568733 46464940384 7360946065403067
147337484 1878083061506 586638705945944130733340117528351711 17609925255617
02135684673650660387863944657037127574045604739023 9244358516828221341
186051134076342405082948659810 7 222685358467615120886903 01 268770501 30953370922
7826 708973217274070021172872383533790896394554702 93247
24050005529633654933765674756910112 3701501

52581061 4247206951199651397422850517961440975433066346093 8 29160522008
3024309219587040244192927942732924294 96 38318532995
729307607921279214068250414568530809617

99677270323142999257318848544590937699656811171778
33125406900938048051264038456703086330402307227605
(98233212297248565708465992251348153470458096 786709
$82351082432439038458781115¢8818962484159655009167
20991765924739327445272769963189214721236405962150
21229789943389240156602836456269171518212005741253 9778591606
70722704018404058e2011627827995849858600 965719769640391809233378925 136442790
18237369256848. £3603105901524408298040967481134471903302405595862
33165611574525593084486588¢53406€378526154619061 77
36010277109607397025842231028342519304740025021091
(44514884521185855343822371163866875376767099001527
843707340407585105257083949707579965271659384949. 3003061
910348902590939347994997549609682328952 7629
27903084600450084125711095857785623559778905220009
95721186036756343925617667236404163126922339753789 2715522 3957946304671
784595069293742820722376042099
$1088613319706992366190189427357097842539630861094
39008095522415348441583846064129419086621135159760
$3492325150745268966018735357249744542162036792251
932741590525455014419581945812127587000799854907 33202204573
75675915437629935526959614301 7993941057 239693863491126609076830249¢2425527341
366896405142 25134987184070428735854177149627702075715673832004
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